QI Lecture 22

Observers

By carefully isolating two quantum systems and inducing one to measure the other (and record the result) we gain some insight into the nature of measurement. Previously we have extrapolated from interactions between isolated systems to understand interactions with the environment. Here we look at interactions between quantum systems and "observers", themselves quantum systems with the ability to record information.

We find, despite the daunting philosophical repercussions, that the rules governing the behavior of isolated quantum systems describe the behavior of observers just as well.

Wigner's Friend Experiment

Wigner's Friend is a thought experiment in the vein of Schrödinger's cat. In this thought experiment, Wigner's friend, W, is the "cat in the box" and they're asked to do a measurement (observe a diagonal photon, count radioactive decays, etc.) on a principle quantum system, P. The system that Wigner's Friend observes is in a superposition of states, so from their perspective the state "collapses" to one of the states in the measurement basis.

However, from Wigner's perspective, both the principle system and his Friend are in a superposition of states. In fact, they're entangled. The question is: does Wigner's Friend collapse the system they're observing, or do they become entangled with it? Does the observation lead to collapse, like this

$$\alpha|\psi\rangle_p|?\rangle_w + \beta|\phi\rangle_p|?\rangle_w \quad \longrightarrow \quad |\psi\rangle_p|``\psi"\rangle_w$$

or entanglement, like this

$$\alpha|\psi\rangle_p|?\rangle_w + \beta|\phi\rangle_p|?\rangle_w \quad \longrightarrow \quad \alpha|\psi\rangle_p|``\psi"\rangle_w + \beta|\phi\rangle_p|``\phi"\rangle_w$$

The quotations in $|``\psi"\rangle_w$ are used to signify that the "**pointer**" system is not in the state $|\psi\rangle$, but is a record of $|\psi\rangle$ being observed.

A not-unreasonable way to define an "**observer**" is as something that can make a verifiable measurement of a quantum system, by encoding the result of that measurement in another physical quantum system. So Alice and Bob are observers not because they're people or often conscious, but because they have a pen and paper.

In this experiment we want to know whether an observation of a test system is objective, in the sense that every other observer in the universe must agree with an observation once it's made, or if it's subjective, meaning that different observers may disagree. We do this by showing that the "principle" and "pointer[1]" quantum systems are entangled and therefore in a superposition.

To date, there are no examples of quantum phenomena that are strictly scale dependent. Entanglement can be established across the planet, coherence can be maintained for minutes at a time, and visible macroscopic objects[2] can be placed in superpositions of states. Now we find that observation, that bottomless source of quantum weirdness, obeys quantum mechanical laws. We find that there's nothing special about the act of measuring and recording information. Even such information processing systems are happy to be in a superposition of states.

This is a profound result with philosophical repercussions that should keep you awake at night.

Just to really draw a line under it: this is not theoretical. The experiment described here is from the paper "Experimental rejection of observer-independence in the quantum world",[3] where they demonstrated their claims to 5 sigma certainty.

Inside Wigner's Lab

In the experiment "Wigner's Lab" is everything inside the dotted box in figure 3. It's a polarizing beam splitter that measures the polarization of the incoming photon, a, without changing it. The result is recorded on qubit c and qubit b is used to announce that a measurement has been successfully done.

The initial state is

$$|\psi\rangle_a|\Psi_-\rangle_{bc} = \left[\alpha|0\rangle_a + \beta|1\rangle_a\right]\left(\frac{|01\rangle_{bc} - |10\rangle_{bc}}{\sqrt{2}}\right) = \frac{\alpha}{\sqrt{2}}|001\rangle - \frac{\alpha}{\sqrt{2}}|010\rangle + \frac{\beta}{\sqrt{2}}|101\rangle - \frac{\beta}{\sqrt{2}}|110\rangle$$

The polarizing beam splitter (PBS) transmits horizontal photons and reflects vertical photons with an extra phase of i. In this experiment, coincidence counters disregard occasions where both photons leave by the same path, so we project onto $Span\{|00\rangle_{ab}, |11\rangle_{ab}\}$.

Taking this selection into account, we can describe the effect of this PBS as

[1] or "record", which is why we're using the subscript r.

[2] Tiny needles about $60\mu m$ long (*barely* visible) were placed in a superposition of vibrational modes. This is demonstrated in "Quantum ground state and single-phonon control of a mechanical resonator", by O'Connell, A. D.; Hofheinz, M.; Ansmann, M.; Bialczak, Radoslaw C.; Lenander, M.; Lucero, Erik; Neeley, M.; Sank, D.; Wang, H.; Weides, M.; Wenner, J.; Martinis, John M.; Cleland, A. N.

[3] By Massimiliano Proietti, Alexander Pickston, Francesco Graffitti, Peter Barrow, Dmytro Kundys, Cyril Branciard, Martin Ringbauer, and Alessandro Fedrizzi

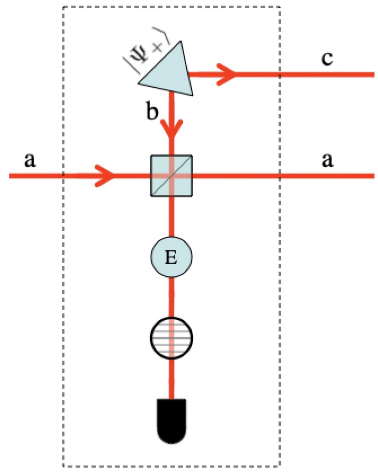

Figure 1: "Wigner's Friend" is a source of polarization-entangled photon pairs, a polarizing beam splitter, and a measurement that selects properly recorded states. This set up maps $|0\rangle_a \to \frac{1}{2}|0\rangle_a|1\rangle_c$ and $|1\rangle_a \to -\frac{1}{2}|1\rangle_a|0\rangle_c$.

$$G = -|00\rangle\langle 00| + |11\rangle\langle 11|$$

That negative is the i^2 collected from the two reflecting horizontal states. The state after the PBS is therefore

$$G_{ab}|\psi\rangle_a|\Psi_-\rangle_{bc} = -\frac{\alpha}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|110\rangle$$

We've left this unnormalized to keep track of the fraction of the original state that has been selected for.[4] In this case, the magnitude squared of the state is $\frac{1}{2}$, so there's a $\frac{1}{2}$ chance of seeing this state and a $\frac{1}{2}$ chance that we'll have some other state, that we'll ignore because it's been rejected by the coincidence counters.

This is a beautiful example of error correction at work. We're selecting only for those states in which a and b are the same and removing all the "error" states where they're not.

Wigner (and the rest of the outside world) need to know that a measurement has successfully been done without discovering the result. This is a perfect job for a quantum eraser. Qubit b is directed through a quarter wave plate aligned at $\frac{\pi}{4}$ followed by a half wave plate aligned at $\frac{\pi}{8}$. We'll denote combination of these operations as E and we find that

[4] This is a "non-trace preserving quantum operation", meaning there's a chance that we'll apply this operation (50% chance of a random selection making this happen) and there's a chance we won't. Also, we have a great opportunity to be lazy by not renormalizing with each step.

$$E = H\left(\frac{\pi}{8}\right)Q\left(\frac{\pi}{4}\right) \sim \left(-\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\right)\left(\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\right) = -\frac{1}{\sqrt{2}}\begin{bmatrix} i & i \\ 1 & -1 \end{bmatrix}$$

which has a useful effect,

$$E|0\rangle = \frac{-i|0\rangle - |1\rangle}{\sqrt{2}} = -i|R\rangle \qquad E|1\rangle = \frac{-i|0\rangle + |1\rangle}{\sqrt{2}} = -i|L\rangle$$

where $|L\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}$ and $|R\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$ are the left and right circular polarization states. So after the quarter and half wave plates, the state of the system is

$$E_b G_{ab}|\psi\rangle_a|\Psi_-\rangle_{bc} = -\frac{\alpha}{\sqrt{2}}|0\rangle_a\left(\frac{-i|0\rangle_b - |1\rangle_b}{\sqrt{2}}\right)|1\rangle_c - \frac{\beta}{\sqrt{2}}|1\rangle_a\left(\frac{-i|0\rangle_b + |1\rangle_b}{\sqrt{2}}\right)|0\rangle_c$$

Before going to a detector, b goes through a horizontal polarizing filter[5], so that the detector only clicks for $|1\rangle_b$. In other words, we select for and execute a projection, $P^{(1)} = |1\rangle\langle 1|$, that leaves the state as

$$P_b^{(1)} E_b G_{ab}|\psi\rangle_a|\Psi_-\rangle_{bc} = \frac{\alpha}{2}|011\rangle - \frac{\beta}{2}|110\rangle$$

Finally, the state leaving Wigner's Lab is

$$|\Omega\rangle = \frac{\alpha}{2}|0\rangle_a|1\rangle_c - \frac{\beta}{2}|1\rangle_a|0\rangle_c$$

where the extra factor of $\frac{1}{2}$ in the amplitude signifies that this state has a one in four chance of existing.

Notice that we've executed a very gentle "observation" in the computational basis. The state of a has been measured and physically recorded into c, such that $|1\rangle_c = |\text{"0"}\rangle_c$ and $|0\rangle_c = |\text{"1"}\rangle_c$. The b photon has done two things: announce the success of that measurement without revealing the result, as well as nailing down the phase relationship between the states. This isn't a given; if the final polarizing filter had been $P_b^{(0)}$, then the state exiting the Lab would be $|\Omega\rangle = \frac{\alpha}{2}|0\rangle_a|1\rangle_c + \frac{\beta}{2}|1\rangle_a|0\rangle_c$.

Outside Wigner's Lab

Outside of Wigner's Lab, we need to choose whether to check that Wigner's Friend accurately did their test, or if Wigner's Friend and their observation are in a superposition.

To check that they've done their job right, we feed in a known state and measure a and c in the computational basis.

[5] In the original experiment they used another polarizing beam splitter to direct the vertical state away from the detector, but the effect is the same.

$$|\psi\rangle_a = |0\rangle_a \longrightarrow |\Omega\rangle_{ac} = \frac{1}{2}|01\rangle_{ac} \qquad\qquad |\psi\rangle_a = |1\rangle_a \longrightarrow |\Omega\rangle_{ac} = -\frac{1}{2}|10\rangle_{ac}$$

The results are always opposites of each other and the a channel remains the same, meaning that Wigner's Friend does accurate measurements.

To check that Wigner's Friend is in a superposition, we input a maximally coherent state

$$|\psi\rangle_a = |\pm\rangle_a = \frac{|0\rangle_a \pm |1\rangle_a}{\sqrt{2}} \qquad \longrightarrow \qquad |\Omega\rangle_{ac} = \frac{1}{2}|\Psi_\mp\rangle_{ac} = \frac{1}{2}\left(\frac{|01\rangle_{ac} \mp |10\rangle_{ac}}{\sqrt{2}}\right)$$

In other words, we can verify that the principle and pointer states are explicitly entangled, by measuring the output in the Bell basis. Evidently, Wigner's Friend is in a superposition of states and they're entangled with the state they're observing.[6] No collapse involved.

Clear as a Bell Test

The standard way to demonstrate quantum behavior[7] is to use a Bell test. In this case, we set up two identical experiments, and send entangled photons into each, with one rotated by $\frac{\pi}{8}$. We do this so that our pairs of measurements are offset from each other, exactly as they were in the Bell test from lecture 11. In this case Alice and Bob can both do the same pairs of measurements, with the $\frac{\pi}{8}$ offset supplied by a polarization rotator as opposed to a rotation of measurement apparatus (which would be difficult in this case).

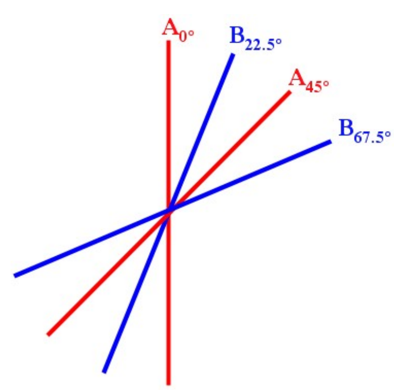

Figure 2: The relative alignments of measurements needed to maximize the CHSH inequality.

The measurement operators we apply at the output are either

[6] One can't help wondering what that feels like (for someone else) to be entangled.

[7] or to at least rule out local realism

$$A_0 = I_a \otimes [|0\rangle_c\langle 0|_c - |1\rangle_c\langle 1|_c]$$

or

$$A_1 = |\Psi_+\rangle_{ac}\langle\Psi_+|_{ac} - |\Psi_-\rangle_{ac}\langle\Psi_-|_{ac}$$

with Bob's measurements, B_0 and B_1, defined in exactly the same way for the output of the second Lab. Although we're not directly comparing the polarizations of two photons, the idea here is exactly the same as we saw in lecture 11.

The ± 1 results from these observables are plugged into the CHSH inequality

$$S = E[a_0 b_0] + E[a_1 b_0] + E[a_1 b_1] - E[a_0 b_1]$$

and once again we find that $S = 2\sqrt{2} > 2$. In other words, because they can't be described using a probability distribution, the principle/pointer space cannot be in a definite (and merely unknown) state.

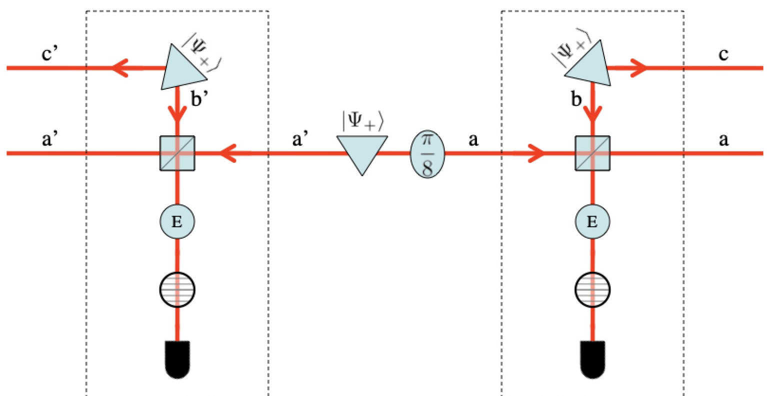

Figure 3: To do a Bell test we compare two measurements on the outputs. Just as with the original CHSH test, each measurement is offset from its partner by $\frac{\pi}{4}$ and the two pairs are offset by $\frac{\pi}{8}$ relative to each other.

As always, there are loopholes that this particular experiment doesn't cover; possible ways for the universe to conspire (on an unnerving scale) to make definite states act like superpositions.

The Rules of Entanglement

Instead of using postulate three, the declared rules for measurement, we have the option of describing measurements as an entanglement between systems.

Orthogonality of Repeatably Measured States

One of the first quantum mechanical rules any physicist learns is that repeated observations produce the same results. In the context here, that means that we need the unitary operator that marks the pointer state to leave the principle system intact. We'll find it interesting to consider two states left intact by the same measurement:

$$U|\psi\rangle_p|?\rangle_r = |\psi\rangle_p|``\psi"\rangle_r \qquad\qquad U|\phi\rangle_p|?\rangle_r = |\phi\rangle_p|``\phi"\rangle_r$$

By unitarity, we can take the inner products of these two equations to get

$$\langle\phi|\psi\rangle_p = \langle\phi|\psi\rangle_p\langle``\phi"|``\psi"\rangle_r$$

which implies that either $\langle\phi|\psi\rangle_p = 0$ or $\langle``\phi"|``\psi"\rangle_r = 1$. Both of these are perfectly valid and worth considering. In the example above, when Alice measured "3 vs. not 3", any two linear combinations of $|0\rangle$ and $|1\rangle$ would be recorded as $|``\not{3}"\rangle$, and yet there's no need for them to be orthogonal to one another.

The other option, where $\langle\phi|\psi\rangle_p = 0$, is more interesting. This says that if the results of the measurements of two states are distinct from each other and repeatable, then those states must be orthogonal. This is yet another beautiful example of orthogonality implying "classical behavior". In this case we find that orthogonal states can be verified repeatably, very much as we expect from classical states.

The structure of measurements are quickly coming into focus. Repeatably measuring a state without changing it implies the application of a projection operator (since $P^2 = P$) and the fact that repeatably measured states corresponding to distinct results are orthogonal implies that these projection operators are themselves orthogonal.

Selection $\longrightarrow$ Projection

We have a heck of a tool at our fingertips. We've seen previously that POVMs can be described as entanglement with a second system followed by a measurement in that system. But this is essentially just the "principle" and "pointer" systems at work. Right off the bat, we find that entanglement looks a lot like measurements.

Example Assume the state

$$|\psi\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle$$

to which Alice does a "three or not three" measurement. The effect can be easily calculated through the projective measurement postulates, but we can also think of measurements as merely labeling the pointer space; a unitary operator defined as

$$U|1\rangle_p|?\rangle_r = |1\rangle_p|\text{“}\not{3}\text{”}\rangle_r \qquad U|2\rangle_p|?\rangle_r = |2\rangle_p|\text{“}\not{3}\text{”}\rangle_r \qquad U|3\rangle_p|?\rangle_r = |3\rangle_p|\text{“}3\text{”}\rangle_r$$

$$\begin{aligned}
& U[\alpha|1\rangle_p + \beta|2\rangle_p + \gamma|3\rangle_p]|?\rangle_r \\
= \; & \alpha|1\rangle_p|\text{“}\not{3}\text{”}\rangle_r + \beta|2\rangle_p|\text{“}\not{3}\text{”}\rangle_r + \gamma|3\rangle_p|\text{“}3\text{”}\rangle_r \\
= \; & [\alpha|1\rangle_p + \beta|2\rangle_p]\,|\text{“}\not{3}\text{”}\rangle_r + \gamma|3\rangle_p|\text{“}3\text{”}\rangle_r
\end{aligned}$$

Notice that if you were to ask Alice the result of her measurement and the state that remains the answers would be

$$[\alpha|1\rangle_p + \beta|2\rangle_p]\,|\text{“}\not{3}\text{”}\rangle_r \qquad or \qquad \gamma|3\rangle_p|\text{“}3\text{”}\rangle_r$$

Up to normalization and an unimportant global phase, these are exactly the states we'd expect from a projective measurement using $P_3 = |3\rangle\langle 3|$ and $P_{\not{3}} = |1\rangle\langle 1| + |2\rangle\langle 2|$. Here we see that the evolution of the system as a whole is strictly unitary, while the perspective of any particular selected state is non-unitary, since it perceives projections.

We already know that, given the distinct pointer states, $|\text{“}\not{3}\text{”}\rangle_r$ and $|\text{“}3\text{”}\rangle_r$, the two selected states, $\alpha|1\rangle_p + \beta|2\rangle_p$ and $\gamma|3\rangle_p$, should be orthogonal (and indeed they are!).

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So repeatably measurable states correspond to the eigenspaces of the projection operators of a measurement.

Linearity implies that the evolution of states attached to a given pointer state are independent.

Spookiness

Although we've used entanglement continuously so far, it's important to stop and consider what the world looks like through entanglement. Consider the most basic example of entanglement. If Alice and Bob share $|\Phi_+\rangle_{ab}$ and haven't done any measurements on it, then the initial state of the system is

$$|?\rangle_\alpha|?\rangle_\beta|\Psi_+\rangle_{ab}$$

where a and b are the principle system, while α and β are the pointer system. If Bob measures his qubit in the computational basis, represented by the operator L_b, the result is

$$L_b|?\rangle_\alpha|?\rangle_\beta\left(\frac{|01\rangle_{ab} + |10\rangle_{ab}}{\sqrt{2}}\right) = |?\rangle_\alpha\left(\frac{|\text{“}1\text{”}\rangle_\beta|01\rangle_{ab} + |\text{“}0\text{”}\rangle_\beta|10\rangle_{ab}}{\sqrt{2}}\right)$$

This isn't too remarkable. We saw the same thing in the Wigner's Friend experiment above; one system measures another and the two become entangled. If Alice now measures her qubit in the computational basis, represented by the operator L_a, the result is

$$L_a L_b |?\rangle_\alpha |?\rangle_\beta \left(\frac{|01\rangle_{ab} + |10\rangle_{ab}}{\sqrt{2}} \right) = \frac{|\text{“}0\text{”}\rangle_\alpha |\text{“}1\text{”}\rangle_\beta |01\rangle_{ab} + |\text{“}1\text{”}\rangle_\alpha |\text{“}0\text{”}\rangle_\beta |10\rangle_{ab}}{\sqrt{2}}$$

Notice that, while we haven't explicitly used the measurement formalism with its implied "wave function collapse" and "spooky action at a distance", both Alice and Bob can be forgiven for thinking they've just seen something spooky. However, no effect of any kind actually passed between them. When they check notes, they'll find that they have gotten opposite results and each of them (and each version) could claim that their own result forced their friend's result. For example, if Alice find herself in the state $|\text{“}0\text{”}\rangle_\alpha$, then she can be confident that when she talks to Bob, he'll be in the state $|\text{“}1\text{”}\rangle_\beta$. The "0 Alice" could no more meet a "0 Bob" than she could meet the other "1 Alice".

Quantum Erasing Bob

Something somewhat more subtle happens when Alice and Bob measure in different bases. By agreeing to do this, Alice and Bob are putting the other into a coherent state; literally, they're mutually quantum erasing each others' measurement results. Note that

$$|\Psi_+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{|0+\rangle - |0-\rangle + |1+\rangle + |1-\rangle}{2}$$

implying that if Alice measures in $\{|0\rangle_a, |1\rangle_a\}$ while Bob measures in $\{|+\rangle_b, |-\rangle_b\}$, then the state of the system afterwards is

$$\frac{1}{2}|\text{“}0\text{”}\rangle_\alpha |\text{“}+\text{”}\rangle_\beta |0+\rangle_{ab} - \frac{1}{2}|\text{“}0\text{”}\rangle_\alpha |\text{“}-\text{”}\rangle_\beta |0-\rangle_{ab} + \frac{1}{2}|\text{“}1\text{”}\rangle_\alpha |\text{“}+\text{”}\rangle_\beta |1+\rangle_{ab} + \frac{1}{2}|\text{“}1\text{”}\rangle_\alpha |\text{“}-\text{”}\rangle_\beta |1-\rangle_{ab}$$

When we looked at this before (lecture 8), we stopped at pointing out that Alice and Bob will not have correlated states. Here we can say a bit more. From Alice's perspective(s) this is

$$\frac{1}{\sqrt{2}}|\text{“}0\text{”}\rangle_\alpha |0\rangle_a \left(\frac{|\text{“}+\text{”}\rangle_\beta |+\rangle_b - |\text{“}-\text{”}\rangle_\beta |-\rangle_b}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}}|\text{“}1\text{”}\rangle_\alpha |1\rangle_a \left(\frac{|\text{“}+\text{”}\rangle_\beta |+\rangle_b + |\text{“}-\text{”}\rangle_\beta |-\rangle_b}{\sqrt{2}} \right)$$

and from Bob's perspective(s) this is

$$\frac{1}{\sqrt{2}}|\text{“}+\text{”}\rangle_\beta |+\rangle_b \left(\frac{|\text{“}0\text{”}\rangle_\alpha |0\rangle_a + |\text{“}1\text{”}\rangle_\alpha |1\rangle_a}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}}|\text{“}-\text{”}\rangle_\beta |-\rangle_b \left(\frac{-|\text{“}0\text{”}\rangle_\alpha |0\rangle_a + |\text{“}1\text{”}\rangle_\alpha |1\rangle_a}{\sqrt{2}} \right)$$

In other words, as with Wigner's Friend's experiment, each sees the other as being in a superposition of states, entangled with their qubit. Notice that the pointer states don't get special treatment; they're in superpositions of states, even relative to each other. Alice, a pointer state, sees Bob, another pointer state, as being in a superposition of states. The question remains, what happens when they meet each other. Unlike when they measured in the same basis, they don't know what they'll see when they meet up to check notes.

We need a way of predicting probabilities, and that means Born's Rule.

Born's Rule

We'll start by showing that states with equal amplitude are equally likely. Laplace did something similar with classical probability when he proposed "When nothing favors any one outcome, symmetry implies they are equiprobable."

Suppose we start with a principle state, $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, and then measure it, recording the result in an entangled pointer state,

$$\frac{1}{\sqrt{2}}|0\rangle_p|\text{``0''}\rangle_r + \frac{1}{\sqrt{2}}|1\rangle_p|\text{``1''}\rangle_r$$

Each of the results, "0" or "1", happen with some probability and not at the same time. If we swap the principle states and then swap the pointer states, we find that nothing has changed

$$\frac{|0\rangle_p|\text{``0''}\rangle_r + |1\rangle_p|\text{``1''}\rangle_r}{\sqrt{2}} \xrightarrow{\textit{swap } p} \frac{|1\rangle_p|\text{``0''}\rangle_r + |0\rangle_p|\text{``1''}\rangle_r}{\sqrt{2}} \xrightarrow{\textit{swap } r} \frac{|1\rangle_p|\text{``1''}\rangle_r + |0\rangle_p|\text{``0''}\rangle_r}{\sqrt{2}}$$

We're still relying on postulate 1 (a physical system is described by its state vector), so the fact that we can swap the state and its record without changing anything implies that the two results are equally likely,[8] $p_0 = p_1 = \frac{1}{2}$. Therefore, we can say that the probabilities are equal for each result of a measurement of $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ done in the computational basis.

We can repeat exactly the same reasoning for any pair of N equal-amplitudes, meaning that $p_k = \frac{1}{N}$, $\forall k$.

We can recover the more general form of Born's rule by "fine-graining" the pointer space.[9] Consider the state $\alpha|0\rangle + \beta|1\rangle$, where $\alpha \propto \sqrt{n}$ and $\beta \propto \sqrt{m}$, and define a family of pointer states

$$|\text{``0''}\rangle_r = \frac{1}{\sqrt{n}}\sum_{k=1}^{n}|\text{``}0_k\text{''}\rangle_r \qquad |\text{``1''}\rangle_r = \frac{1}{\sqrt{m}}\sum_{j=1}^{m}|\text{``}1_j\text{''}\rangle_r$$

[8] This isn't a given: try repeating this with $\frac{\sqrt{3}|0\rangle+|1\rangle}{2}$.

[9] We did something similar when we derived Shannon Entropy in lecture 7.

and imagine a new "file cabinet space" (another place to keep records) correlated with the pointer space such that a measurement/entanglement yields

$$[\alpha|0\rangle_p + \beta|1\rangle_p]|?\rangle_r|?\rangle_f \quad \longrightarrow \quad \sum_{k=1}^{n}|0\rangle_p|\text{``}0_k\text{''}\rangle_r|\text{``}0_k\text{''}\rangle_f + \sum_{j=1}^{m}|1\rangle_p|\text{``}1_j\text{''}\rangle_r|\text{``}1_j\text{''}\rangle_f$$

This looks completely silly, but the idea is that we can now swap the principle and pointer spaces together, followed by the file cabinet space, in order to show equiprobability again. Therefore, the probability of each of the two results is proportional to the number of terms in these two sums:

$$p(0) = \frac{n}{n+m} = |\alpha|^2 \qquad\qquad p(1) = \frac{m}{n+m} = |\beta|^2$$

since we have $\alpha \propto \sqrt{n}$, $\beta \propto \sqrt{m}$, and $|\alpha|^2 + |\beta|^2 = 1$. More generally, the squaring of the amplitudes derives from the Pythagorean theorem, which is a property of Hilbert spaces (postulate 1).

Example We can extend this result to linear combinations of states, like the ones from the earlier example

$$[\alpha|1\rangle_p + \beta|2\rangle_p]\,|\text{``}\not{3}\text{''}\rangle_r \qquad or \qquad \gamma|3\rangle_p|\text{``}3\text{''}\rangle_r$$

Immediately we see that the probability of observing a 3 (of being $|\text{``}\not{3}\text{''}\rangle_r$) is $|\gamma|^2$. The probability of the other state, $\alpha|1\rangle + \beta|2\rangle$, isn't quite obvious. However, this state is nothing special. We can just define it as a basis state, $\xi|1'\rangle$, and since the coordinate transformation, U, is unitary we can calculate the magnitude of ξ.

$$\begin{aligned}
\xi^*\xi\langle 1'|1'\rangle &= [\alpha^*\langle 1| + \beta^*\langle 2|]\,U^\dagger U\,[\alpha|1\rangle + \beta|2\rangle] \\
|\xi|^2 &= [\alpha^*\langle 1| + \beta^*\langle 2|]\,[\alpha|1\rangle + \beta|2\rangle] \\
|\xi|^2 &= |\alpha|^2 + |\beta|^2
\end{aligned}$$

That's Pythagorus in a Hilbert space again. What we've just found is that the probability of $|\eta\rangle = \alpha|1\rangle + \beta|2\rangle$ is $\langle\eta|\eta\rangle$. Or, given that $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle$,

$$p(3) = \left[\langle\psi|P_3^\dagger\right][P_3|\psi\rangle] = \langle\psi|P_3|\psi\rangle \qquad\qquad p(\not{3}) = \left[\langle\psi|P_{\not{3}}^\dagger\right][P_{\not{3}}|\psi\rangle] = \langle\psi|P_{\not{3}}|\psi\rangle$$

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The Rules of Measurement

In very short, the measurement postulates for projective measures (from lecture 16) are:

- 1-2) Measurements are a set of projection operators such that $\sum_m P_m = I$.
- 3) A measurement of $|\psi\rangle$ yields one outcome, m.
- 4) $p(m) = \langle\psi|P_m|\psi\rangle$
- 5) The state after a measurement is $|\psi_m\rangle = \frac{P_m|\psi\rangle}{\sqrt{p(m)}}$.
- 6) Unreported measurements produce a mixed state $\rho = \sum_m p(m)|\psi_m\rangle\langle\psi_m|$.

1-3 come from the orthogonality of repeated measurements. Repeatability ensures that the operators are projections, and orthogonality ensures that the results are distinguishable.

4 is Born's rule, which we just derived.

5 is another way of stating the fact that the component of the principle state that remains is the part corresponding to a particular pointer state.

6 is about how to deal with regular probabilities (ignorance) rather than quantum mechanics (see lecture 6). If you don't know which state you have, you can still figure out the probability of particular occurrences by adding up all the possible states weighted by their probabilities.

Some Philosophy

The geocentric theory, where the Earth is at the center of the solar system, feels right. After all, the Earth is clearly not moving (according to all of our animal senses). But there are issues with the geocentric theory. When we look at the Galilean moons of Jupiter, or even our own Moon, we find that they obey simple, elegant, and extremely precise physical laws (Newton's laws of motion and gravitation) and when we look at the other planets we find that they weave and loop across the sky. To describe this motion "epicycles" were invented; essentially just Fourier series used to approximate the path.

The problem with this is that we had to come up with excuses for why Jupiter's moons behaved one way, while Jupiter itself behaved another. It takes a lot of force to move planets around and there didn't seem to be anything responsible for doing it. The solution turned out to be putting all matter on the same footing: the Sun is in the center (it turned out) because it's about a thousand times more massive than everything else put together, and the anomalous motion of the other planets was just an observer effect caused by the motion of the Earth itself. Every object and planet, including the most important one,[10] obeys the exact same set of <u>universal</u> laws.

All that said, if you want to find Saturn in the sky, epicycles are a much better way to figure where to point your telescope. The full Newtonian (or even better, general relativistic) treatment is a lot of work. You have to keep track of lots of things that you don't really need, like where Earth is, or how massive Saturn is, or how far apart everything is, or the warping of the coordinate system. Epicycles are ultimately inaccurate and don't represent anything that physically exists, but they're an efficient tool for backyard astronomers who just want to know where to look. Add up a few vectors on a few circles: easy.

Wave function collapse is a similar idea. We find that isolated quantum systems follow a set of remarkably simple physical laws. By assuming that we and the universe at large exist in a definite state, we force quantum mechanics to "contort" in ways that are not generally feasible. Wave function collapse is inexplicable, requires the invention of new phenomena, resists rudimentary investigation, and somehow directly contradicts and supersedes the quantum mechanical laws established for isolated systems.

What we have with Wigner's Friend is an entanglement framework for observation that is the application of known physical laws and which doesn't (as far as we know) require us to introduce new phenomena. And while it is ontologically more satisfying, that doesn't necessarily mean it's more useful. When you use entanglement to model measurement, you have to keep track of a lot of information that you shouldn't care about, such as the "non-realized states" and their amplitudes. Wave function collapse retains only that information that is pertinent to ourselves; the probability that you, personally, will see a particular result and the state of your system given that result.

If you don't care what other, distinct states of you are doing, you can rest assured that they don't care what you're doing either. Not that there's any reason to worry about those other versions, but if you do worry about them, then they're probably worried about you too.

And... Consciousness?

Here's what can be said. The definition of "observer" that we're using here, "*something that can make a verifiable measurement of a quantum system, by encoding the result of that measurement in another physical quantum system*" is extremely broad. The combination of circuit elements from the experiment above serves as an observer, because it measures one quantum system and verifiably records the result into another.

Devices like geiger counters or single-photon detectors are physicals machines that explicitly relay the distinct results of measurements on quantum systems. A mind can

[10] It's Earth, unless someone has a very compelling argument otherwise.

be used to store information about those results, so in precisely that sense a person is an observer. And so is a chalkboard.